Future Muon Dipole Moment Measurements

B. Lee Roberts*

Department of Physics, Boston University

590 Commonwealth Avenue

Boston, MA 02215 USA

From the famous experiments of Stern and Gerlach to the present, measurements of magnetic dipole moments, and searches for electric dipole moments of "elementary" particles have played a major role in our understanding of sub-atomic physics. In this talk I discuss the progress on measurements and theory of the magnetic dipole moment of the muon. I also discuss a new proposal to search for a permanent electric dipole moment (EDM) of the muon and put it into the more general context of other EDM searches. These experiments, along with searches for the lepton flavor violating decays $\mu \to e\gamma$ and $\mu^- + A \to e^- + A$, provide a path to the high-energy frontier through precision measurements.

1. Introduction and theory of the lepton anomalies

Over the past 83 years, the study of dipole moments of elementary particles has provided a wealth of information on subatomic physics. From the pioneering work of Stern[1] through the discovery of the large anomalous magnetic moments of the proton[2] and neutron[3], the ground work was laid for the discovery of spin, of radiative corrections and the renormalizable theory of QED, of the quark structure of baryons and the development of QCD.

A charged particle with spin $\vec{s}$ has a magnetic moment

$$\vec{\mu}_s = g_s(\frac{e}{2m})\vec{s};\ a \equiv \frac{(g_s - 2)}{2};\ \mu = (1+a)\frac{e\hbar}{2m};\ (1)$$

where g_s is the gyromagnetic ratio, a is the anomaly, and the latter expression is what one finds in the Particle Data Tables.[4]

The Dirac equation tells us that for spin one-half point-like particles, $g \equiv 2$ for spin angular momentum, and is unity for orbital angular momentum (the latter having been verified experimentally[6]). For point particles, the anomaly arises from radiative corrections, two examples of which are shown in Fig. 1. The lowest-order correction gives the famous Schwinger[5] result, $a = \alpha/2\pi$, which was verified experimentally by Foley and Kusch.[6] The situation for baryons is quite different, since their internal quark structure gives them large anomalies.

In general a (or g) is an expansion in $\left(\frac{\alpha}{\pi}\right)$,

$$a = C_1\ \left(\frac{\alpha}{\pi}\right) + C_2\ \left(\frac{\alpha}{\pi}\right)^2 + C_3\ \left(\frac{\alpha}{\pi}\right)^3 + C_4\ \left(\frac{\alpha}{\pi}\right)^4 + \cdots (2)$$

with 1 diagram for the Schwinger (second-order) contribution, 5 for the fourth order, 40 for the sixth order, 891 for the eighth order.

The QED contributions to electron and muon $(g - 2)$ have now been calculated through eighth order, $(\alpha/\pi)^4$ and the tenth-order contribution has been estimated.[8]
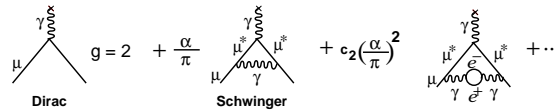

Figure 1. A schematic of the first few terms in the QED expansion for the muon. The vacuum polarization term shown is one of five of order $(\alpha/\pi)^2$.

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	-	+	+
C	-	-	-
T	+	-	-

Table 1
Transformation properties of the magnetic and electric fields and dipole moments.

Particle	Present Limit (e-cm)	SM Value (e-cm)
n[20]	6.3×10^{-26}	$< 10^{-31}$
e^-[21]	$\sim 1.6 \times 10^{-27}$	$< 10^{-38}$
μ[19]	$< 10^{-18}$ (CERN) $\sim 10^{-19}$ (E821)* $\sim 10^{-24}$ J-PARC†	$< 10^{-35}$

Table 2
Measured limits on electric dipole moments, and their standard model values.
* Estimated limit, work in progress.
†Letter of Intent (LOI) to J-PARC for a new dedicated experiment.[22]

While magnetic dipole moments (MDMs) are a natural property of charged particles with spin, electric dipole moments (EDMs) are forbidden both by parity *(P)* and by time reversal *(T)* symmetries.[9,10,11] This can be seen by examining the Hamiltonian for a spin one-half particle in the presence of both an electric and magnetic field, $\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$. The transformation properties of $\vec{E}$, $\vec{B}$, $\vec{\mu}$ and $\vec{d}$ are given in Table 1, and we see that while $\vec{\mu} \cdot \vec{B}$ is even under all three, $\vec{d} \cdot \vec{E}$ is odd under both *P* and *T*. Thus the existence of an EDM implies that both *P* and *T* are violated. In the context of *CPT* symmetry, an EDM implies *CP* violation. The standard model value for the electron and muon EDMs are well beyond the reach of experiment (see Table 2), so observation of a non-zero *e* or μ EDM would be a clear signal for new physics. Since the presently known *CP* violation is inadequate to describe the baryon asymmetry in the universe, additional sources of *CP* violation should be present. Furthermore, we do expect to find *CP* violation in the lepton sector. New dynamics such as supersymmetry could easily produce new sources of *CP* violation which could have a possible connection with cosmology (leptogenesis).[12,13,14]

The magnetic and electric dipole moments can be represented as the real and imaginary parts of a generalized dipole operator *D*, and the interaction Lagrangian becomes

$$\mathcal{L}_{dm} = \frac{1}{2}\left[D\bar{\mu}\sigma^{\alpha\beta}\frac{1+\gamma_5}{2} + D^*\bar{\mu}\sigma^{\alpha\beta}\frac{1-\gamma_5}{2}\right]\mu F_{\alpha\beta} \quad (3)$$

with Re $D = a_\mu \frac{e}{2m_\mu}$ and Im $D = d_\mu$.

The electron anomaly is now measured to a relative precision of about four parts in a billion (ppb),[16] which is better than the precision on the fine-structure constant α, and Ki-

noshita has used the measured electron anomaly to give the best determination of α.[17] The electron anomaly will be further improved over the next few years.[18]

The muon anomaly is measured to 0.5 parts per million (ppm).[23,24,25] The relative contributions of heavier particles to *a* scales as $(m_e/m_\mu)^2$, so the muon has an increased sensitivity to higher mass scale radiative corrections of about 40,000 over the electron. At a precision of ~ 0.5 ppm, the muon anomaly is sensitive to ≥ 100 GeV scale physics.

The standard model value of a_μ has measurable contributions from three types of radiative processes: QED loops containing leptons (e, μ, τ) and photons;[8] hadronic loops containing hadrons in vacuum polarization loops;[26,27,28,29,30,31,32, 33,34,35,36,37] and weak loops involving the *W* and *Z* weak gauge bosons (the standard model Higgs contribution is negligible),[26]

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Had}) + a_\mu(\text{Weak}). \quad (4)$$

A significant difference between the experimental value and the standard model prediction would signify the presence of new physics. A few examples of such potential contributions are lepton substructure, anomalous $W - \gamma$ couplings, and supersymmetry.[26]

The CERN experiment[19] observed the contribution of hadronic vacuum polarization shown in Fig. 2(a) at the 8 standard deviation level. Unfortunately, the hadronic contribution cannot be

calculated directly from QCD, since the energy scale is very low ($m_\mu c^2$), although Blum[38] has performed a proof of principle calculation on the lattice. Fortunately dispersion theory gives a relationship between the vacuum polarization loop and the cross section for $e^+e^- \to$ hadrons,

$$a_\mu(\text{Had};1) = (\frac{\alpha m_\mu}{3\pi})^2 \int_{4m_\pi^2}^\infty \frac{ds}{s^2} K(s)R(s), \qquad (5)$$

where

$$R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \to \mu^+\mu^-)} \qquad (6)$$

and experimental data are used as input. The factor s^{-2} in the dispersion relation, means that values of $R(s)$ at low energies (the ρ resonance) dominate the determination of $a_\mu(\text{Had};1)$. In principle, this information could be obtained from hadronic τ^- decays such as $\tau^- \to \pi^-\pi^0\nu_\tau$, which can be related to e^+e^- annihilation through the CVC hypothesis and isospin conservation.[27,28,29,30] However, inconsistencies between information obtained from e^+e^- annihilation and hadronic tau decays, plus an independent confirmation of the CMD2 high-precision e^+e^- cross-section measurements by the KLOE collaboration,[39] have prompted Davier, Höcker, et al, to state that until these inconsistencies can be understood only the e^+e^- data should be used to determine $a_\mu(\text{Had};1)$.[32]
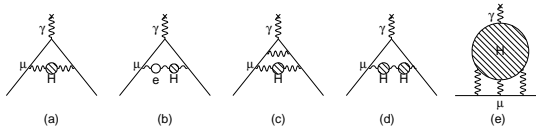

Figure 2. The hadronic contribution to the muon anomaly, where the dominant contribution comes from (a). The hadronic light-by-light contribution is shown in (e).

The hadronic light-by-light contribution (see Fig. 2(e)) has been the topic of much theoretical
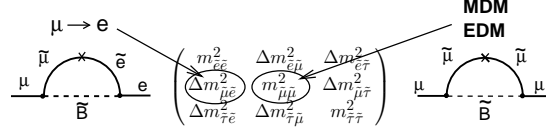

Figure 3. The supersymmetric contributions to the anomaly, and to $\mu \to e$ conversion, showing the relevant slepton mixing matrix elements. The MDM and EDM give the real and imaginary parts of the matrix element, respectively.

investigation.[33,34,35,36,37] Unlike the lowest-order contribution, it can only be calculated from a model, and this contribution is likely to provide the ultimate limit to the precision of the standard-model value of a_μ.

One of the very useful roles the measurements of a_μ have played in the past is placing serious restrictions on physics beyond the standard model. With the development of supersymmetric theories as a favored scheme of physics beyond the standard model, interest in the experimental and theoretical value of a_μ has grown substantially. SUSY contributions to a_μ could be at a measurable level in a broad range of models. Furthermore, there is a complementarity between the SUSY contributions to the MDM, EDM and transition moment for the lepton-flavor violating (LFV) process $\mu^- \to e^-$ in the field of a nucleus. The MDM and EDM are related to the real and imaginary parts of the diagonal element of the slepton mixing matrix, and the transition moment is related to the off diagonal one, as shown in Fig. 3. This reaction, along with the companion LFV decay $\mu^+ \to e^-\gamma$, will be searched for in "next generation" experiments now under construction.[41,42]

From neutrino oscillations we already know that lepton flavor is violated, and this violation will be enhanced if there is new dynamics at the TeV scale. This same new physics could also generate measurable effects in the magnetic and electric dipole moments of the muon as well.[40,12,13,14]

2. Measurement of the muon anomaly

The method used in the third CERN experiment and the BNL experiment are very similar, save the use of direct muon injection[43] into the storage ring,[44,45] which was developed by the E821 collaboration. These experiments are based on the fact that for $a_\mu > 0$ the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field. The spin precession frequency ω_S consists of the Larmor and Thomas spin-precession terms. The spin frequency ω_S, the momentum precession (cyclotron) frequency ω_C, are given by

$$\omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}; \qquad \omega_C = \frac{eB}{mc\gamma}. \qquad (7)$$

The difference frequency

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc} \qquad (8)$$

is the frequency with which the spin precesses relative to the momentum, and is proportional to the anomaly, rather than to g. A precision measurement of a_μ requires precision measurements of the muon spin precession frequency ω_a, and the magnetic field, which is expressed as the free-proton precession frequency ω_p in the storage ring magnetic field.

The muon frequency can be measured as accurately as the counting statistics and detector apparatus permit. The design goal for the NMR magnetometer and calibration system was a field accuracy of 0.1 ppm. The B which enters in Eq. 8 is the average field seen by the ensemble of muons in the storage ring. In E821 we reached a precision of 0.17 ppm in the magnetic field measurement.

An electric quadrupole[46] is used for vertical focusing, taking advantage of the "magic" $\gamma = 29.3$ at which an electric field does not contribute to the spin motion relative to the momentum. With both an electric and a magnetic field, the spin difference frequency is given by

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right], \quad (9)$$

which reduces to Eq. 8 in the absence of an electric field. For muons with $\gamma = 29.3$ in an electric

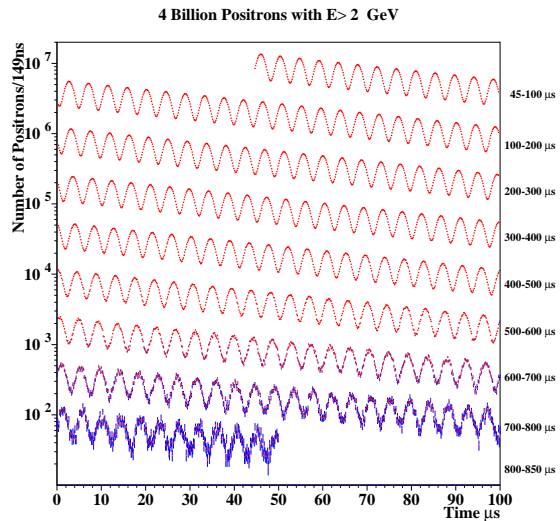field alone, the spin would follow the momentum vector.

Figure 4. The time spectrum of positrons with energy greater than 2.0 GeV from the year 2000 run. The endpoint energy is 3.1 GeV. The time interval for each of the diagonal "wiggles" is given on the right.

The experimental signal is the $e^\pm$ from $\mu^\pm$ decay, which were detected by lead-scintillating fiber calorimeters.[47] The time and energy of each event was stored for analysis offline. Muon decay is a three-body decay, so the 3.1 GeV muons produce a continuum of positrons (electrons) from the end-point energy down. Since the highest energy $e^\pm$ are correlated with the muon spin, if one counts high-energy $e^\pm$ as a function of time, one gets an exponential from muon decay modulated by the $(g-2)$ precession. The expected form for the positron time spectrum is $f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$, however in analyzing the data it is necessary to take a number of small effects into account in order to obtain a satisfactory χ^2 for the fit.[24,25] The data from our 2000 running period is shown in Fig. 4

The experimental results from E821 are shown in Fig. 5, with the average

$$a_\mu(\text{E821}) = 11\,659\,208(6) \times 10^{-10} \qquad (\pm 0.7\,\text{ppm})(10)$$

which determines the "world average". The theory value[32,31]

$$a_\mu(\text{SM}) = 11\,659\,182(6) \times 10^{-10} \qquad (\pm 0.7\,\text{ppm})(11)$$

is taken from Höcker et al.,[32], which updates their earlier analysis[30] with the KLOE data;[39] and from Hagiwara, et al.,[31] who use a different weighting scheme for the experimental data when evaluating the dispersion integral but do not include the KLOE data. When this theory value is compared to the standard model value using either of these two analyses[32,31] for the lowest-order hadronic contribution, one finds

$$\Delta a_\mu(\text{E821} - \text{SM}) = (26 \pm 9.4) \times 10^{-10}, \qquad (12)$$

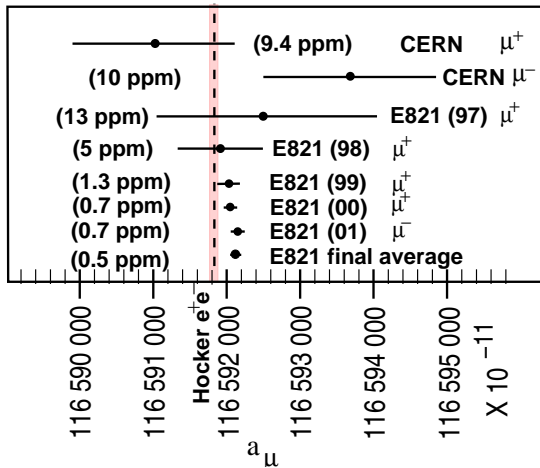or a discrepancy of 2.7 standard deviations.

Figure 5. Measurements of a_μ. The strong interaction contribution is taken from references [31] and [32].

To show the sensitivity of our measurement of a_μ to the presence of virtual electroweak gauge

bosons, we subtract off the electroweak contribution of $15.4(0.1)(0.2) \times 10^{-10}$ from the standard model value, compare with experiment and obtain

$$\Delta a_\mu = (40.7 \pm 8.6) \times 10^{-10}. \qquad (13)$$

a 4.7 standard deviation discrepancy. This difference shows conclusively that E821 was sensitive to physics at the 100 GeV scale. At present, it is inconclusive whether we see evidence for contributions from physics beyond the standard-model gauge bosons.

With each data set, the systematic error was reduced, and for the final data set taken in 2001 the systematic error on a_{μ^-} was 0.27 ppm with a statistical error of 0.66 ppm. Given the tantalizing discrepancy between our result and the latest standard-model value, and the fact that the hadronic error could be reduced by about a factor of two over the next few years,[26] we submitted a new proposal to Brookhaven to further improve the experimental measurement. The goal of this new experiment is ± 0.2 ppm total error, with the goal of controlling the total systematic errors on the magnetic field and on the muon frequency measurement to 0.1 ppm each.

Our proposal[48] was given enthusiastic scientific approval in September 2004 by the Laboratory, and has been given the new number, E969. Negotiations are underway between the Laboratory and the funding agencies to secure funding.

A letter of intent (LOI) for an even more precise $(g - 2)$ experiment was also submitted to J-PARC.[49] In that LOI we proposed to reach a precision below 0.1 ppm. Since it is not clear how well the hadronic contribution can be calculated, and whether the new Brookhaven experiment E969 will go ahead, we will evaluate whether to press forward with this experiment at a later time. Our LOI at J-PARC[49] was predicated on pushing as far as possible at Brookhaven before moving to Japan.

3. The Muon EDM

While the MDM has a substantial standard model value, the predicted EDMs for the leptons are unmeasurably small and lie orders of

magnitude below the present experimental limits given in Table 2. An EDM at a measurable level would signify physics beyond the standard model. SUSY models, and other dynamics at the TeV scale do predict EDMs at measurable levels.[12,13,14,15]

A new experiment to search for a permanent EDM of the muon with a design sensitivity of 10^{-24} e-cm is being planned for J-PARC.[22] This sensitivity lies well within values predicted by some SUSY models.[14] Feng, et al.,[15] have calculated the range of ϕ_{CP} available to such an experiment, assuming a new physics contribution to a_μ of 3×10^{-9},

$$d_\mu^{\mathrm{NP}} \simeq 3 \times 10^{-22} \left(\frac{a_\mu^{\mathrm{NP}}}{3 \times 10^{-9}} \right) \tan \phi_{CP} \ \ e{-}\mathrm{cm} \quad (14)$$

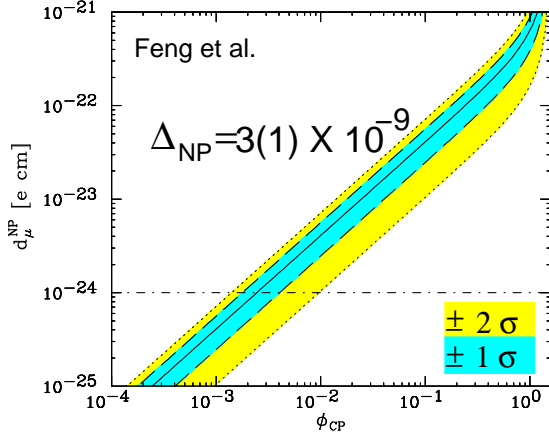where ϕ_{CP} is a CP violating phase. This range is shown in Fig. 6.

Figure 6. The range of ϕ_{CP} available to a dedicated muon EDM experiment.[15] The two bands show the one and two standard-deviation ranges if a_μ differs from the standard model value by $(3 \pm 1) \times 10^{-9}$.

Of course one wishes to measure as many EDMs as possible to understand the nature of the interaction. While naively the muon and electron EDMs scale linearly with mass, in some theories the muon EDM is greatly enhanced relative to linear scaling relative to the electron EDM when the heavy neutrinos of the theory are non-degenerate.[12,13]
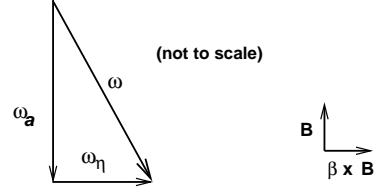

Figure 7. A sketch showing the relationship between ω_a and ω_η.

With an EDM present, the spin precession relative to the momentum is given by

$$\vec{\omega} = -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$
$$+ \frac{e}{m} \left[\frac{\eta}{2} \left(\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right) \right] \quad (15)$$

where

$$d_\mu = \frac{\eta}{2} \left(\frac{e\hbar}{2mc} \right) \simeq \eta \times 4.7 \times 10^{-14} \ \ e{-}\mathrm{cm} \quad (16)$$

and $a_\mu = (\frac{g-2}{2})$. For reasonable values of β, the motional electric field $\vec{\beta} \times \vec{B}$ is much larger than electric fields that can be obtained in the laboratory, and the two vector frequencies are orthogonal to each other.

The EDM has two effects on the precession: the magnitude of the observed frequency is increased, and the precession plane is tipped relative to the magnetic field, as illustrated in Fig. 7. E821 was operated at the magic γ so that the focusing electric field did not cause a spin precession. In E821 the tipping of the precession plane is very small, ($\eta/2a_\mu \simeq 9$ mrad) if one uses the CERN limit[19]

given in Table 2. This small tipping angle makes it very difficult to observe an EDM effect in E821, since the $(g - 2)$ precession (ω_a) is such a large effect.

We have recently introduced a new idea which optimizes the EDM signal, and which uses the motional electric field in the rest frame of the muon interacting with the EDM to cause spin motion.[50] The dedicated experiment will be operated off of the magic γ, for example at ~ 500 MeV/c, and will use a radial electric field to stop the $(g-2)$ precession.[50] Then the spin will follow the momentum as the muons go around the ring, except for any movement arising from an EDM. Thus the EDM would cause a steady build-up of the spin out of the plane with time. Detectors would be placed above and below the storage region, and a time-dependent up-down asymmetry R would be the signal of an EDM,

$$R = \frac{N_{\text{up}} - N_{\text{down}}}{N_{\text{up}} + N_{\text{down}}}. \qquad (17)$$

A simulation for $d_\mu = 2 \times 10^{-20}$ e cm is given in Fig. 8.

The figure of merit for statistics in the EDM experiment is the number of muons times the polarization. In order to reach 10^{-24} e cm, the muon EDM experiment would need $NP^2 \simeq 5 \times 10^{16}$, a flux only available at a future facility. While progress can still be made at Brookhaven on a_μ, a dedicated muon EDM experiment must be done elsewhere.

4. Summary and Conclusions

Measurements of the muon and electron anomalies played an important role in our understanding of sub-atomic physics in the 20th century. The electron anomaly was tied closely to the development of QED. The subsequent measurement of the muon anomaly showed that the muon was indeed a "heavy electron" which obeyed QED.[19] With the sub-ppm accuracy now available for the muon anomaly,[23,24,25] there may be indications that new physics is beginning to appear in loop processes.[51]

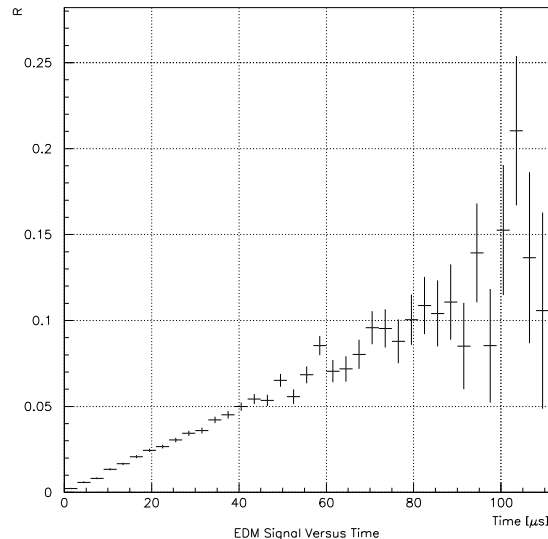The non-observation of an electron EDM is becoming an issue for supersymmetry, just as the

Figure 8. Simulation of an EDM signal for $\eta = 5 \times 10^{-7}$ or $d_\mu = 2.3 \times 10^{-20}$

non-observation of a neutron EDM implies such a mysteriously (some would say un-naturally) small θ-parameter for QCD. The search for EDMs will continue, and if one is observed, the motivation for further searches in other systems will be even stronger. The muon presents a unique opportunity to observe an EDM in a second-generation particle, where the CP phase might be different from the first generation, or the scaling with mass might be quadratic rather than linear.

In closing I consider two scenarios: (i) the LHC finds SUSY; and (ii) the LHC finds the standard model Higgs at a reasonable mass and nothing else.

If SUSY turns out to be *the* extension to the standard model, then there will be SUSY enhancements to a_μ to the muon EDM and also to the amplitudes for lepton flavor violating muon decays. Once the SUSY mass spectrum is measured, a_μ will provide a very clean measurement of $\tan \beta$.[26]

If SUSY or other new dynamics at the TeV scale are not found at LHC, then precision experiments, which are sensitive through virtual loops

8

to much higher mass scales than direct searches for new particles, become even more important. Experiments such as EDM searches, $(g-2)$ and searches for lepton flavor violation, all carried out at high intensity facilities, may provide the only way to probe these higher energy scales.

Opportunities at future high intensity facilities are actively being pursued, and both the theoretical and experimental situations are evolving. It is clear that the study of lepton moments and lepton flavor violation, along with neutron EDM searches, will continue to be a topic of great importance in the first part of the 21st century.

REFERENCES

1. O. Stern, Z. Phys. **7**, 249 (1921), W. Gerlach and O. Stern, Ann. Physik, **74**, 1924.
2. R. Frisch and O. Stern, Z. Phys.**85**, 4 (1933).
3. Luis W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).
4. Particle Data Group, Phys. Lett. **B 592**, Part 1 (2004).
5. J. Schwinger, Phys. Rev. **73**, 416L (1948).
6. H.M. Foley and P. Kusch, Phys. Rev. **73**, 4121 (1948).
7. J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, (1964), p172.
8. T. Kinoshita and M. Nio, Phys. Rev. Lett. **90**, 021803-1, Toichiro Kinoshita and M. Nio, Phys. Rev. **D70**, 113001 (2004).
9. E.M. Purcell and N.F. Ramsey, Phys. Rev. **78**, 807 (1950)
10. L. Landau, Nucl. Phys. **3**, 127 (1957).
11. N.F. Ramsey Phys. Rev. **109**, 225 (1958).
12. J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B **528**, 86 (2002).
13. J. R. Ellis and M. Raidal, Nucl. Phys. B **643** 229, (2002).
14. K.S. Babu, B. Datta, and R.N. Mohapatra, Phys. Rev. Lett.**85**, 5064 (2000).
15. J.L. Feng, K.T. Matchev .Y, Shadmi, Nucl. Phys. **B 613**, 366 (2001), and Phys. Lett. **B555**, 89 (2003).
16. R.S. Van Dyck et al., Phys. Rev. Lett., **59**, 26(1987) and in *Quantum Electrodynamics*, (Directions in High Energy Physics Vol. 7) T. Kinoshita ed., World Scientific, 1990, p.322.
17. T. Kinoshita, Rep. Prog. Phys.**59**,1459 (1996) which is updated in Ref. [8].
18. G. Gabrielse and collaborators have constructed a new trap, and expect to improve on the electron anomaly by a factor of 10 to 100 over the measurements of [16]
19. J. Bailey, et. al, Nucl. Phys. **B150**, 1 (1979).
20. P.G. Harris, et al., Phys. Rev. Lett. **82**, 904 (1999).
21. B.C. Regan, et al., Phys. Rev. Lett. **88**, 071805-1 (2002).
22. J-PARC Letter of Intent L22, *Search for a permanent muon electric dipole moment at the 10^{-24} e cm level.*, Y. Kuno, J.Miller, Y. Semertzidis spokespersons.
23. H.N. Brown, et al., (Muon $(g-2)$ Collaboration), Phys. Rev. Lett. **86** (2001) 2227.
24. G.W. Bennett, et al., (Muon $(g-2)$ Collaboration), Phys. Rev. Lett. **89**, 101804 (2002)
25. G.W. Bennett, et al., (Muon $(g-2)$ Collaboration), Phys. Rev. Lett. **92**, 161802 (2004).
26. M. Davier and W. Marciano, Ann. Rev. Nucl. Part. Sci. **54**, 115 (2004).
27. R. Alemany, M. Davier and A. Höcker, Eur.Phys.J. **C2** 123 (1998).
28. M. Davier and A. Höcker, Phys. Lett.**B419**, 419(1998). Phys. Lett.**B435**, 427 (1998).
29. M. Davier, S. Eidelman, A. Höcker, and Z. Zhang, Eur. Phys J. **C 27** 497 (2003).
30. M. Davier, S. Eidelman, A. Höcker, and Z. Zhang, Eur. Phys. J. **C 31**, 503 (2003).
31. K. Hagiwara, A.D. Martin, Daisuke Nomura, and T. Teubner, Phys. Lett. **B557**, 69 (2003), and Phys. Rev. **D69** 093003 (2004).
32. A. Höcker, ICHEP04, which can be found at http://ichep04.ihep.ac.cn/program.htm, also M. Davier 8th International Workshop on Tau-Lepton Physics, September 2004, http://www.hepl.phys.nagoya-u.ac.jp/public/Tau04/
33. Johan Bijnens, Elisabetta Pallante and Joaquím Prades, Phys. Rev. Lett. **75**,

1447,(1995), Phys. Rev. Lett. **75**, 3781(1995), Nucl. Phys. **B474**, 379 (1996) and Nucl. Phys. **B626**, 410 (2002).

34. M. Hayakawa and T. Kinoshita Phys. Rev. **D57**, 465(1998) and hep-ph/0112102 (2002).

35. Marc Knecht, Andreas Nyffeler, Phys. Rev. **D65**, 073034 (2002), M. Knecht, A. Nyffeler, M. Perrottet, E. De Rafael, Phys. Rev. Lett. **88**, 071802 (2002).

36. I. Blokland, A. Czarnecki and K. Melinkov Phys. Rev. Lett. **88**, 071803 (2002).

37. K. Melnikov and A. Vainshtein, Phys. Rev. **D70**, 113006 (2004).

38. T. Blum, Phys. Rev. Lett. **91**, 052001-1 (2003).

39. A. Aloisio, et al., (KLOE Collaboration) arXiv hep-ex/0407048, July 2004, and Phys. Lett. **B** in press.

40. R. Barbieri and L.J. Hall, Phys. Lett. **B338**, 212 (1994), R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. **B445**, 219 (1995), J. Hisano and D. Nomura, Phys. Rev. **D59**, 116005 (1999).

41. The MEG ($\mu^+ \rightarrow e^+\gamma$) Experiment at the Paul Scherrer Institute is under construction and will collect their first data in 2006. A. Baldini and T. Mori spokespersons. http://meg.web.psi.ch/

42. The MECO Experiment at the Brookhaven AGS, BNL AGS E940, W. Molzon, spokesperson. Single event sensitivity is 10^{-17}. http://meco.ps.uci.edu/

43. E. Efstathiadis, et al., Nucl. Inst. and Meth. **A496**,8-25 (2002).

44. G.T. Danby, et al., Nucl. Instr. and Methods, **A 457**, 151-174 (2001).

45. A. Yamamoto, et al., Nucl. Inst. and Meth. **A496**,8-25 (2002).

46. Y.K. Semertzidis, et al., Nucl. Inst. and Meth. **A503**, 458-484 (2003).

47. S.A. Sedyk, et al., Nucl. Inst.and Meth., **A455** 346, (2000).

48. *A $(g - 2)_\mu$ Experiment to ± 0.2 ppm Precision* R.M. Carey, A. Gafarov, I. Logashenko, K.R. Lynch, J.P. Miller, B.L. Roberts (co-spokesperson), G. Bunce, W. Meng, W.M. Morse (resident spokesperson), Y.K. Semertzidis, D. Grigoriev, B.I. Khazin, S.I. Redin, Yuri M. Shatunov, E. Solodov, Y. Orlov, P. Debevec, D.W. Hertzog (co-spokesperson), P. Kammel, R. McNabb, F. Mülhauser, K.L. Giovanetti, K.P. Jungmann, C.J.G. Onderwater, S. Dhamija, T.P. Gorringe, W. Korsch, F.E. Gray, B. Lauss, E.P. Sichtermann, P. Cushman, T. Qian, P. Shagin, S. Dhawan and F.J.M. Farley, which can be found at: http://g2pc1.bu.edu/~roberts/

49. J-PARC Letter of Intent L17, *An Improved Muon $(g - 2)$ Experiment at J-PARC*, B.L. Roberts contact person.

50. F.J.M. Farley, et al., Phys. Rev. Lett. **92**, 052001 (2004).

51. W.J. Marciano, J. Phys. **G29**, 23 (2003), and J. Phys. **G29**, 225 (2003).